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Lattice Homomorphisms on L^p-Spaces

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Abstract

Let (Ω, Σ, μ) be a complete σ-finite measure space and let $1 \leq p \leq \infty$. We study bounded lattice homomorphisms $T : L^p(\mu) \to F$ into a Banach lattice F. To avoid the obstruction $1 \notin L^1(\mu)$ when $\mu(\Omega) = \infty$, we work on the δ-ring $\Sigma_f = \{A \in \Sigma : \mu(A) < \infty\}$ and the dense sublattice S_f of simple functions with finite-measure support. For $1 \leq p < \infty$ we show that every bounded lattice homomorphism induces a local Boolean set function $\nu : \Sigma_f \to F_+$ via $\nu(A) = T(\chi_A)$, and that T is the unique bounded extension of the associated simple-function integral I_ν from S_f to $L^p(\mu)$. We introduce a p-variation functional $\|\nu\|_{(p)}$ and prove the intrinsic norm identity $\|T\| = \|\nu\|_{(p)}$ together with a converse construction theorem. When F is an AL-space, boundedness is characterised by a Radon–Nikodým derivative $g \in L^q(\mu)$ and $\|T\| = \|g\|_q$. For $p = 1$ on finite measure spaces we also present an order-integral (Kantorovich–Wright type) formulation and show it agrees with the norm-density approach. Finally, we treat the case $p = \infty$ under σ-order continuity.

Keywords: Lattice homomorphisms, Boolean δ-ring measures, p-variation functional, L^p-spaces

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